Node-Based Optimal Routing in Wireless Networks

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Outline

Motivation and Network Model

Cost Function

Optimality Conditions

Routing Algorithm

Summary

Motivation

Centralized algorithms rely on a designated node which

- gathers information about network from other nodes,
- obtains full knowledge of the network's topology, and
- makes routing decisions which are re-distributed to other nodes.

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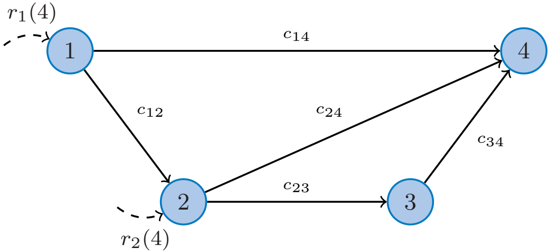
Node-Based algorithms do not employ a special node and therefore

- don't have a single point of failure,
- allow each node to make local routing decision, and
- often show better scaling behavior.

Motivation and Network Model

Network $N = (\mathcal{V}, \mathcal{A})$ consists of

- nodes $\mathcal{V} = \{1, 2, 3, 4\}$,
- arcs $\mathcal{A} = \{(i, k) \mid i, k \in \mathcal{V} \wedge c(i, k) > 0\}$,
- link capacities $\mathcal{C} = \{c_{ik} \mid \forall (i, k) \in \mathcal{A}\}$,
- input rates $r_i(j)$ at node i destined for sink j.



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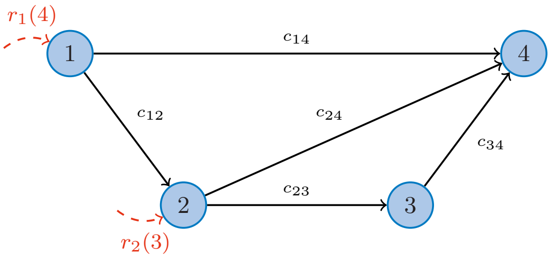

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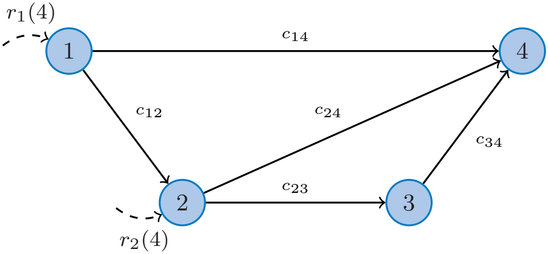

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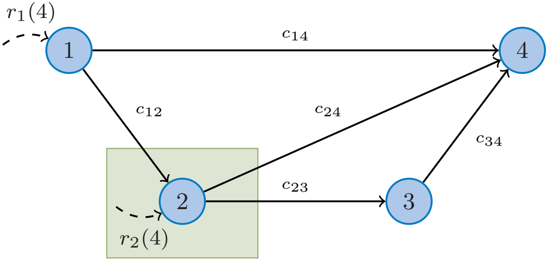

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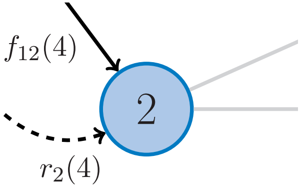

Total expected flow destined for sink j at node i is comprised of

- flow $f_{ki}(j)$ sent to i from k over incoming link (k, i) and
- additional flow $r_i(j)$ generated at node i.

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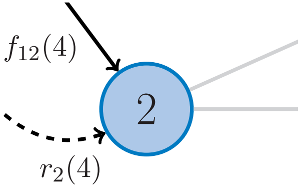

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$$t_i(j) = r_i(j) + \sum_{(k,i)\in\mathcal{A}} f_{ki}(j), \quad \forall j \in \mathcal{V}$$

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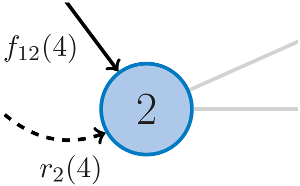

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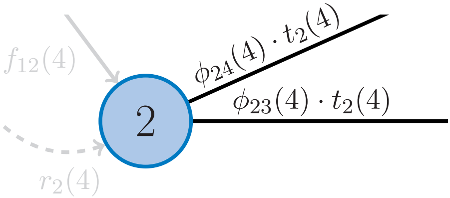
Next question: How is the flow $t_i(j)$ distributed to outgoing links?

Motivation and Network Model



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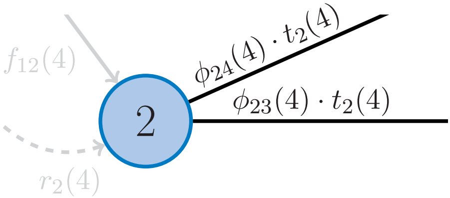


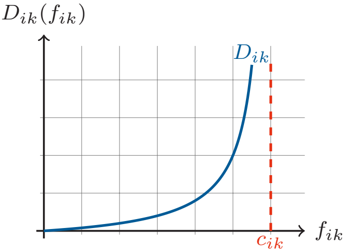
Flow $f_{ik}(j)$ towards j over link (i,k) is determined as fraction of expected traffic $t_i(j)$ according to routing variables $\phi_{ik}(j) \in \Phi$:

- $\phi_{ik}(j) = \frac{f_{ik}(j)}{t_i(j)} \geq 0, \quad \forall j \in \mathcal{V}, \ \forall (i,k) \in \mathcal{A}$,
- $\phi_{ik}(i) = 0, \quad \forall (i,k) \in \mathcal{A}$,
- $\sum_{(i,k) \in \mathcal{A}} \phi_{ik}(j) = 1, \quad \forall j \in \mathcal{V}$.

Outline

Cost Function

- Link flow $f_{ik} = \sum_{j \in \mathcal{V}} f_{ik}(j)$
- Link delay $D_{ik}(f_{ik})$,
  convex and increasing in f_{ik}
- Total delay $D_T = \sum_{(i,k) \in \mathcal{A}} D_{ik}(f_{ik})$



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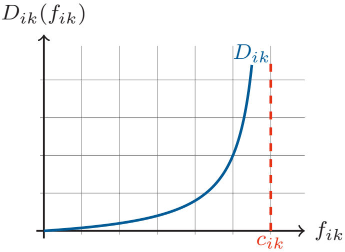


Optimization problem

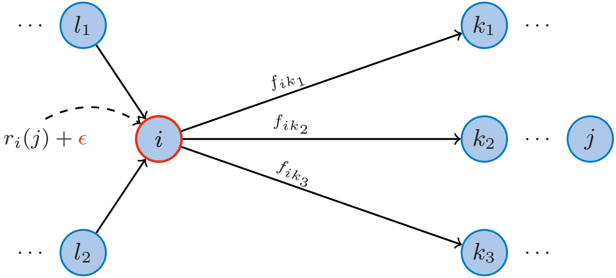
$$\min_{\phi} D_T = \min_{\phi} \sum_{(i,k) \in \mathcal{A}} D_{ik}(f_{ik})$$

s.t. $\phi \in \Phi,\ f_{ik} \leq c_{ik}\ \forall (i,k) \in \mathcal{A}$

Outline

Optimality Conditions

Consider a small increase ϵ of input rate $r_i(j)$:



$$\frac{\partial D_T}{\partial r_i(j)} =$$

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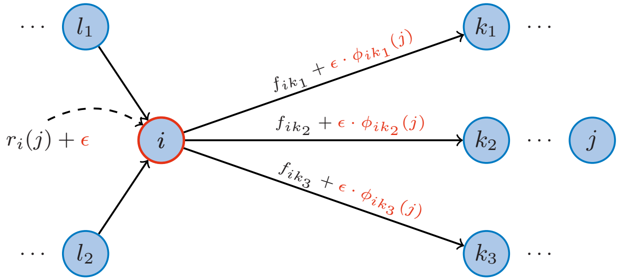

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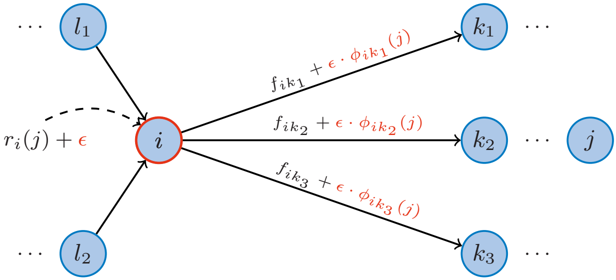

$$\frac{\partial D_T}{\partial r_i(j)} = \sum_{(i,k)\in\mathcal{A}} \phi_{ik}(j) \frac{dD_{ik}}{df_{ik}} +$$

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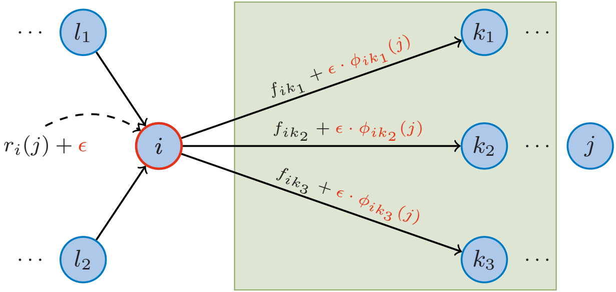

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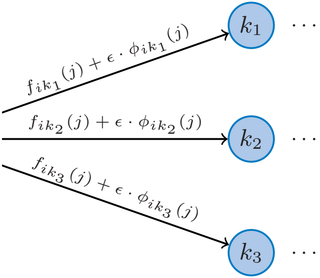

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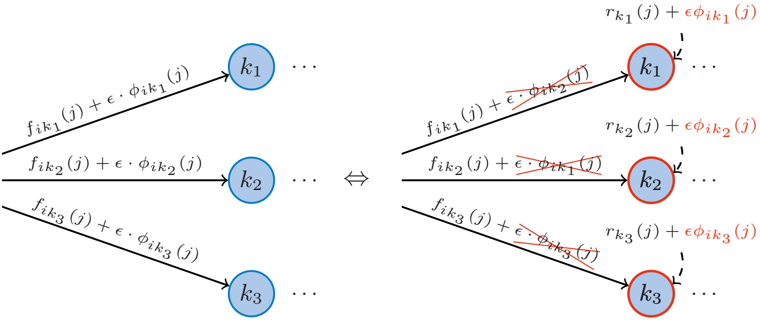

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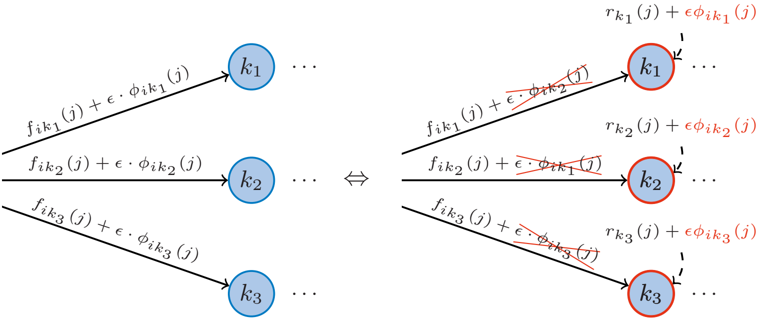
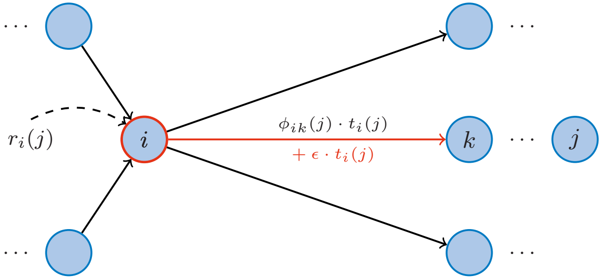


$$\frac{\partial D_T}{\partial r_i(j)} = \sum_{(i,k)\in\mathcal{A}} \phi_{ik}(j)\frac{dD_{ik}}{df_{ik}} + \sum_{(i,k)\in\mathcal{A}} \phi_{ik}(j)\frac{\partial D_T}{\partial r_k(j)}$$

Optimality Conditions

Next, consider a small increment ϵ in routing variable $\phi_{ik}(j)$:



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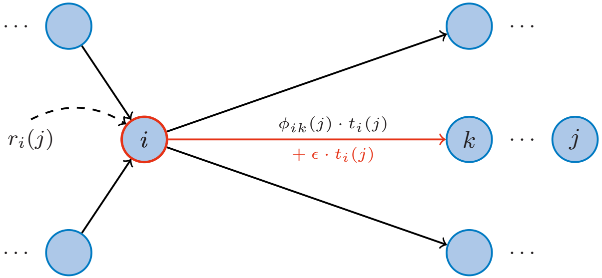

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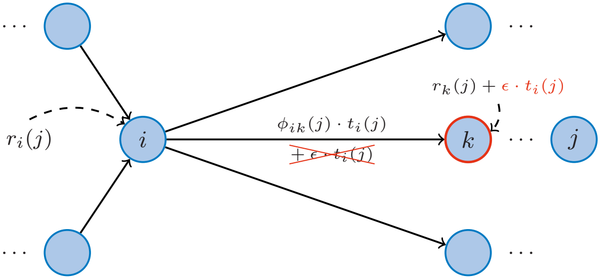

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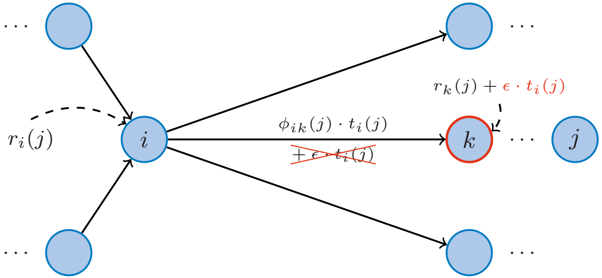

$$\frac{\partial D_T}{\partial \phi_{ik}(j)} = t_i(j) \frac{dD_{ik}}{df_{ik}} + t_i(j) \frac{\partial D_T}{\partial r_k(j)}$$

Optimality Conditions

Change of delay with respect to

- change of input rates

$$\frac{\partial D_T}{\partial r_i(j)} = \sum_{(i,k)\in\mathcal{A}} \phi_{ik}(j) \left[\frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)}\right]$$

- change of routing variables

$$\frac{\partial D_T}{\partial \phi_{ik}(j)} = t_i(j) \cdot \left[\frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)}\right]$$

Optimality Conditions

Necessary condition (derived via KKT)

$$\frac{\partial D_T}{\partial \phi_{ik}(j)} \begin{cases} = \lambda_{ij}, & \phi_{ik}(j) > 0 \\ \geq \lambda_{ij}, & \phi_{ik}(j) = 0 \end{cases}, \ \forall i \neq j, \ \forall (i,k) \in \mathcal{A}$$

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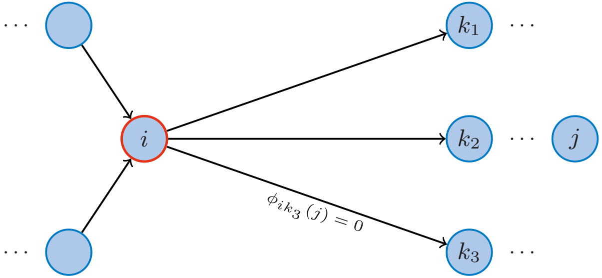

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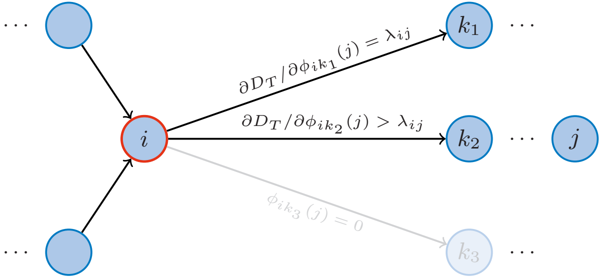

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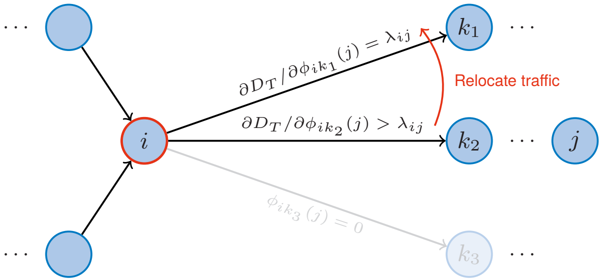

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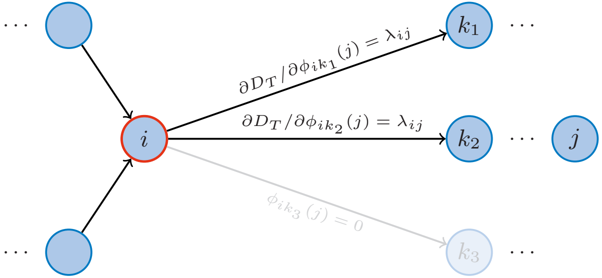

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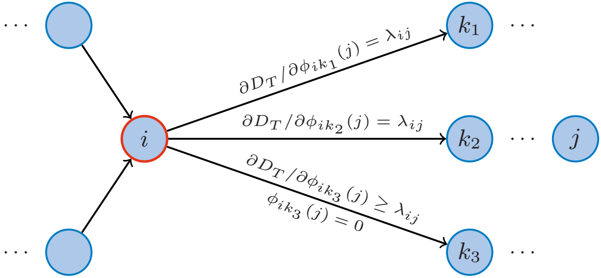

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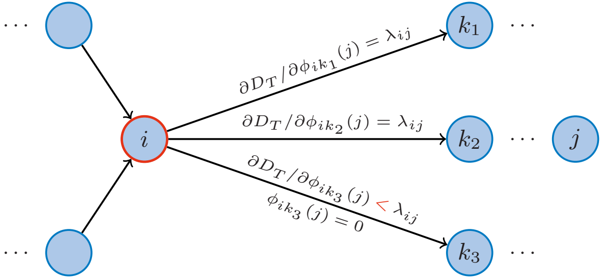

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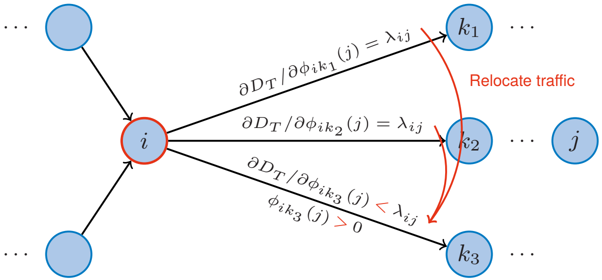

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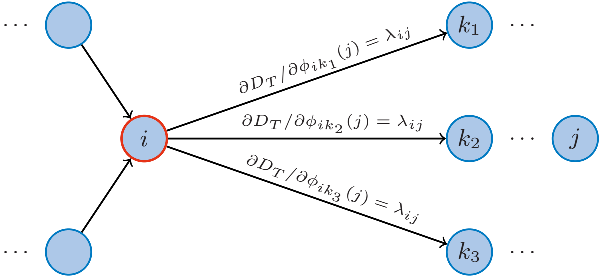
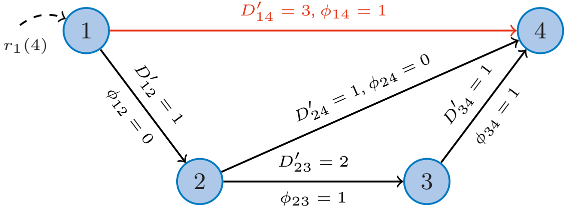


Optimality Conditions

Necessary, but not sufficient

$$\frac{\partial D_T}{\partial \phi_{ik}(j)} = t_i(j) \cdot \left[\frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)} \right] \begin{cases} = \lambda_{ij}, & \phi_{ik}(j) > 0 \\ \geq \lambda_{ij}, & \phi_{ik}(j) = 0 \end{cases}$$

Assume only node 1 generates traffic destined for node 4:



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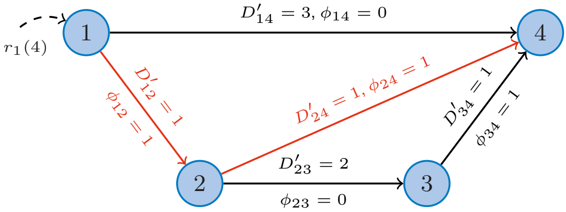

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Therefore, we find that for all $i \neq j$

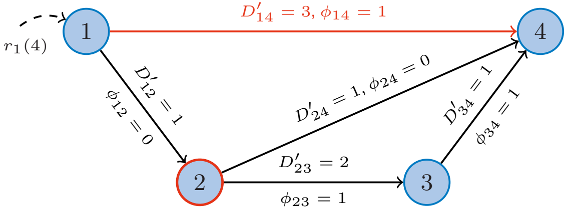
$$\frac{\partial D_T}{\partial r_i(j)} = \sum_{(i,k)\in\mathcal{A}} \phi_{ik}(j) \left[\frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)} \right]$$

$$\begin{cases} = \dfrac{dD_{ik}}{df_{ik}} + \dfrac{\partial D_T}{\partial r_k(j)}, & \text{if } \forall (i,k) \in \mathcal{A}:\ \phi_{ik}(j) > 0, \\ \leq \dfrac{dD_{ik}}{df_{ik}} + \dfrac{\partial D_T}{\partial r_k(j)}, & \text{if } \exists (i,k) \in \mathcal{A}:\ \phi_{ik}(j) = 0. \end{cases}$$

Optimality Conditions

Sufficient condition

$$\frac{\partial D_T}{\partial r_i(j)} \leq \frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)}, \ \forall i \neq j, \ \forall (i,k) \in \mathcal{A}$$



$$\frac{\partial D_T}{\partial r_2(4)} = \phi_{24}(4)\left[\frac{dD_{24}}{df_{24}} + \frac{\partial D_T}{r_4(4)}\right] + \phi_{23}(4)\left[\frac{dD_{23}}{df_{23}} + \frac{\partial D_T}{r_3(3)}\right] = 3$$

$$\frac{dD_{24}}{df_{24}} + \frac{\partial D_T}{r_4(4)} = 1$$

Optimality Conditions

Sufficient condition

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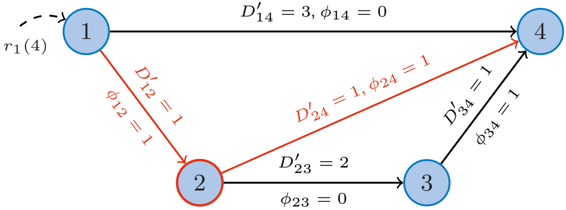

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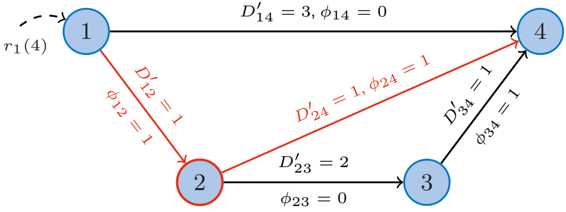
Sufficient condition

$$\frac{\partial D_T}{\partial r_i(j)} \leq \frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)}, \; \forall i \neq j, \; \forall (i,k) \in \mathcal{A}$$



Proof: Skipped (technical), can be found in: *Robert G. Gallager, A Minimum Delay Routing Algorithm Using Distributed Computation*.

Outline

Routing Algorithm

We consider an iterative routing algorithm consisting of two phases:

1. Propagation of delay gradients to upstream neighbors
2. Update of routing variables according to information received from downstream neighbors

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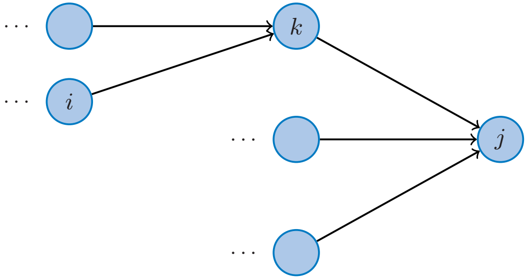
For the following example assume that

- only one flow exists (traffic destined for sink j) and
- the network is loop-free.

Both assumptions ease notation and can be relaxed
(networks with loops require some extensions to the algorithm).

Routing Algorithm

Phase 1: Propagation of delay gradients to upstream neighbors



Routing Algorithm

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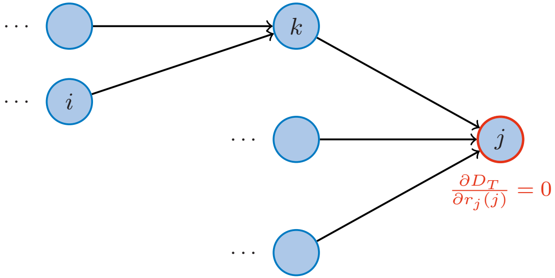

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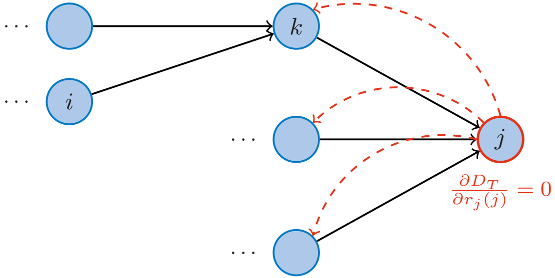

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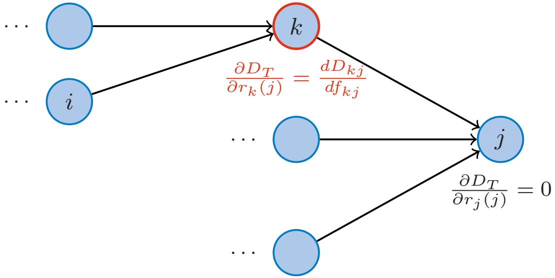

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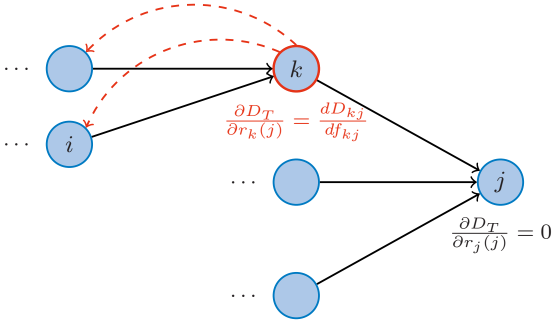

Routing Algorithm

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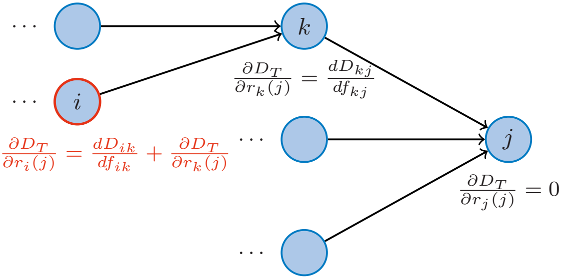


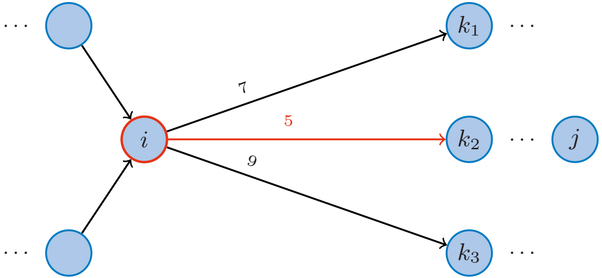
Routing Algorithm

Phase 2: Update of routing variables

Each node $i \in \mathcal{V}$ performs the following steps:

1. Choose the best link (i, m_i) with

$$m_i = \underset{\{k \mid (i,k) \in \mathcal{A}\}}{\arg\min} \left[\frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)} \right].$$



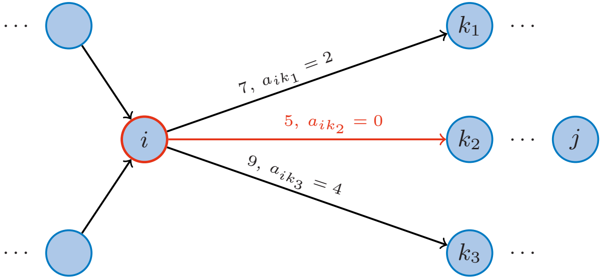
Routing Algorithm

Phase 2: Update of routing variables

Each node $i \in \mathcal{V}$ performs the following steps:

2. Between any link (i, k) and the best link calculate the difference of delay slopes

$$a_{ik}(j) := \frac{dD_{ik}}{df_{ik}} + \frac{\partial D_T}{\partial r_k(j)} - \left[\frac{dD_{im_i}}{df_{im_i}} + \frac{\partial D_T}{\partial r_{m_i}(j)}\right].$$



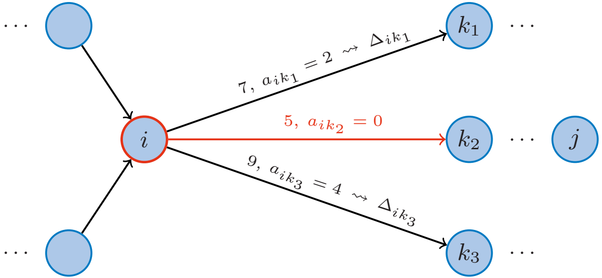
Routing Algorithm

Phase 2: Update of routing variables

Each node $i \in \mathcal{V}$ performs the following steps:

3. For each routing variable $\phi_{ik}(j)$ determine correction term

$$\Delta_{ik}(j) := \min\left\{\phi_{ik}(j), \frac{a_{ik}(j)}{t_i(j)}\right\}.$$



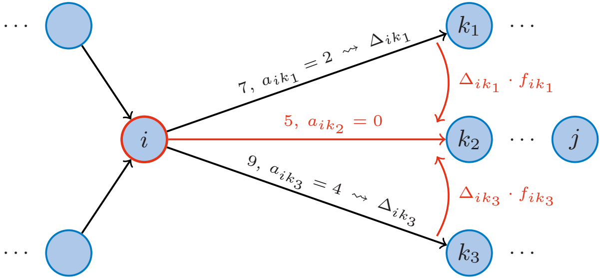
Routing Algorithm

Phase 2: Update of routing variables

Each node $i \in \mathcal{V}$ performs the following steps:

4. Update routing variables for step n:

$$\phi_{ik}(j)^{(n)} = \begin{cases} \phi_{ik}^{(n-1)}(j) - \Delta_{ik}(j), & k \neq m_i, \\ \phi_{ik}^{(n-1)}(j) + \sum\limits_{k \neq m_i} \Delta_{ik}(j), & k = m_i. \end{cases}$$



Outline

Summary

We saw an

- iterative routing algorithm
- operating in a distributed manner and
- converging to a minimum-delay solution

by usage of delay information from downstream neighbors.

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- operating in a distributed manner and
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by usage of delay information from downstream neighbors.

It can be adapted to allow for

- networks with loops ($\rightarrow$ blocking set),
  (Robert G. Gallager, A Minimum Delay Routing Algorithm Using Distributed Computation), and
- power control ($\rightarrow$ power control and allocation variables),
  (Yufang Xi and Edmund Yeh, Node-Based Optimal Power Control, Routing, and Congestion Control in Wireless Networks).